**Exercise 1.** *Let $I$ be an interval. Given continuous function $f(x)$ defined on $I$, $f(I)$ has a maximum and minimum. Then, $I$ is compact.*

*Proof.* Remind that $f(x) = x$ is a continuous function. In addition, $f(I) = I$. Since $f(I)$ is compact, $f(I) = I$ is bounded. So, we have four possible cases $I = (a, b), (a, b], [a, b), [a, b]$. In any case, $\inf f(I) = a$ and $\sup f(I) = b$. Hence, $a, b \in I$. Namely, $I = [a, b]$ is compact. □

**Exercise 2.** *Let $S$ be a set in $\mathbb{R}$. Given continuous function $f(x)$ defined on $S$, $f(S)$ is a compact interval. Then, $S$ is compact.*

*Proof.* Remind that $f(x) = x$ is a continuous function. In addition, $f(S) = S$. Since $f(S)$ is a compact interval, $S$ is a compact interval. □

**Exercise 3.** *Let $S = [-1, -2] \cup [1, 2]$ and $f(x) = x^2$. Then, $f(S) = [1, 4]$ is a compact interval. Namely, even if $S$ is disconnected and $f(x)$ is continuous, $f(S)$ can be an interval.*